Pratice Problems for Final Exam

1. (i). State the definition of Möbius transformations.

(ii). Find a Möbius transformation that maps the unit circle $\mathcal{S} = \{|z| = 1\}$ to the real axis.

(iii). Find a Möbius transformation that maps the unit circle $\mathcal{S} = \{|z| = 1\}$ to the real line $x = y$.

2.(i) State the definition of a branch of logarithm on a region G.

(ii). Let $\mathcal{S}$ be the unit circle. Prove there is no branch of logarithm on any region containing $\mathcal{S}$.

3. (i). Let γ be a closed, rectifiable curve, and a a point not on the curve γ. State the definition of the index $n(\gamma; a)$ of γ with repect to a.

(ii). Prove $n(\gamma; a)$ must be an integer.

4. (i) State the Cauchy's estimate theorem.

(ii). State the Liouville's theorem.

(iii). Use the Liouville's theorem to prove the fundamental theorem of algebra.

5. Let G be a region with $a \in G$ and $f : G \to \mathbb{C}$ a nonconstant analytic function. Assume $|f(a)| \leq |f(z)|$ for all $z \in G$. Prove $f(a) = 0$.

6. Let f be an analytic function on the unit disc $B(0;1)$. Assume $|f(z)| \leq 1$ for $|z| < 1$. Show $|f'(0)| \leq 1$.

7. Each of the following fuctions f has an isolated singularity at $z = 0$. Determine its nature (removable, pole or esstential singularity). Justify your answer.

(a). $\frac{\sin z}{z}$;

(b). $\frac{\cos z}{z}$;

(c). $\frac{\cos z - 1}{z}$;

(d). $e^{\frac{1}{z}}$;

(e). $z \sin \frac{1}{z}$.

8. All homework problems and especially the graded ones.